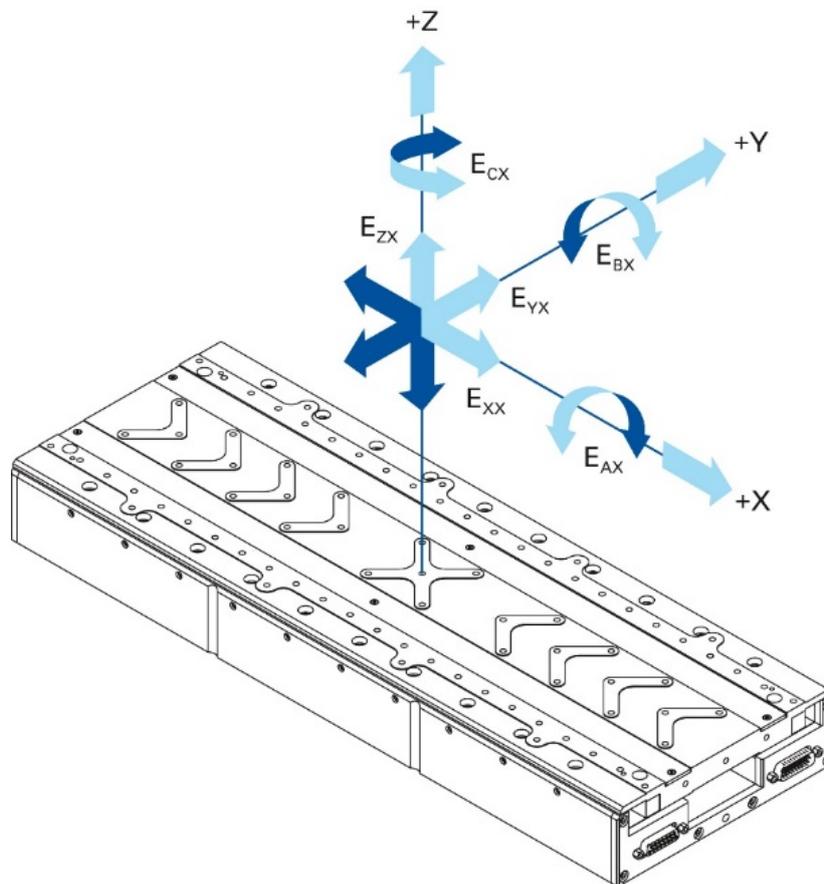


## This Is How PI Does Measuring - Part II

Positioning Accuracy, Repeatability, and similar Measurements: Measurement Procedure and Data Preparation at PI



# 1 Qualification of Positioning Axes

Measurements are carried out on prototypes, individual products and series-production products to qualify the positioning axes produced by PI (Physik Instrumente). In the case of series-production products, valid specifications are derived based on these measured values and then published in the datasheets. These specifications describe the fundamental positioning characteristics for a product type, such as e.g. positioning accuracy, angular error in multiple axes, and unidirectional and bidirectional repeatability.

It is important particularly in the design of customer-specific solutions that there is a common understanding of what the specifications are because it is only in this way that the design can be clearly defined.

This document describes

- which measurements are carried out and which measurement methods are used,
- which standards are considered to this process,
- how the data acquired from the measurements is prepared,
- how measurement results are derived from the data and what terminology is used to describe them.

In addition, this document provides a brief overview of the measurement protocol and the measurement results that are reported and how both are structured.

## 2 Applicable Standards and Special Features of Precision Positioning

The measurement rules described here are mainly based on the ISO 230 series of international standards, whereby the standards ISO 230-1 (2012-03) and ISO 230-2 (2014-05) are primarily relevant, as well as the American standard ASME B5.54 (2005). This standard is to a large extent identical to the

ISO 230 series of standards with respect to the measurements described in this document.

A major disadvantage of the stated standards for the qualification of micropositioning and nanopositioning systems is the fact that they were actually designed for larger turning lathes, milling machines, and similar products. Therefore, the explanations and detailed descriptions for the methods described in these standards are not always applicable for the qualification of products that are produced for high-precision micropositioning and nanopositioning. If it was not possible to transfer the ideas from these standards directly to the qualification of PI products, the measurement rules described in these standards were modified.

This means that the measurement rules used at PI are not a direct copy of the stated standards but rather they include specific adaptations and, in particular, enhancements of them with the aim of being able to test micropositioning and nanopositioning axes in the best way possible.

PI actively takes part in the current ASME B5.64 standardization committee which works on the development of a measuring standard for micro and nanopositioning systems.

## 3 Basic Measurement Procedure and the Measuring Equipment Used

### 3.1 General Measurement Procedure

The positioning axes produced by PI are designed and specified for a defined travel range.



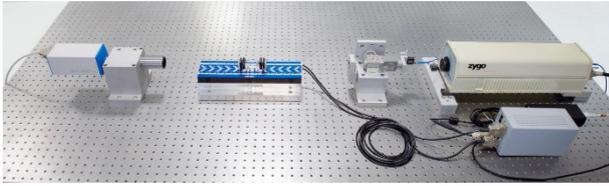


Fig. 2 A typical measurement setup, from left: an autocollimator to measure angular errors, a positioning axis on which two reflecting mirrors are mounted, and an interferometer setup to measure the displacement.

This type of setup is shown in Figure 2. An interferometer is used from one direction to determine the linear displacement of the axis being tested, while an autocollimator is used to determine the angular error from the opposite direction at the same time. The positioning axis is equipped with two reflector mirrors, which are used by the interferometer and autocollimator for their measurements. The measuring equipment used by PI is described in more detail in the Whitepaper “This is how PI Does Measuring - Part I”.

### 3.3 Correction of the Raw Data

#### 3.3.1 Correction of the Abbe Error

An Abbe error occurs when an angular error has caused an error in the linear measurement (e.g. positioning accuracy).

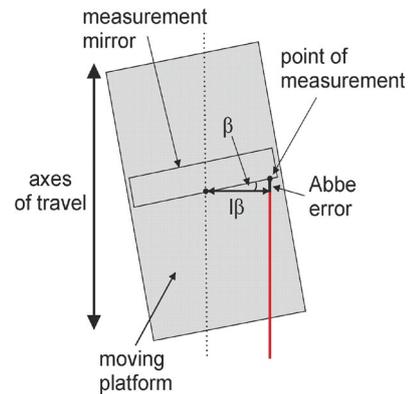
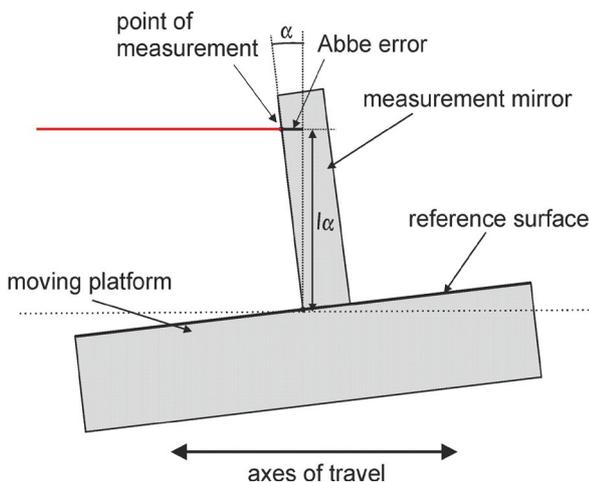


Fig. 3 Abbe errors for both angles  $\alpha$ ,  $\beta$ : red = measuring beam,  $l_\alpha$  and  $l_\beta$  describe the distance to the axes with respect to the interferometer's measuring point. A correction is made to arrive at a point located at the surface of the moveable platform.

This is illustrated in Figure 3: Due to the tilt of the moveable platform, there is an angular error when the beam from the interferometer hits the measurement mirror. The effect on the linear measurement value depends on the distance between the mirror and the platform. There is thus an error in the measurement value that can be corrected accordingly and eliminated from the data.

Depending on the setup of the measuring equipment relative to the moveable platform, there can be an Abbe error in one or two directions.

To calculate the correction, the linear measurement values are adjusted based on the angular data measured using the autocollimator. The distance between the moveable mirror and the surface of the moveable platform is determined and used for the calculation. This distance is also stated in the measurement protocol. Therefore, the measurement protocol shows whether or not a tilt correction of the measurement data has been carried out.

The angular and linear measurements are carried out separately from one another at PI. This means that the linear and angular errors can be distinguished from one another and are reported separately.

If an Abbe error has been corrected during the test procedure, the surface of the moveable platform is used as the reference surface. The correction is calculated based on this surface.

Using this information, the resulting positioning error for points at any distance to the platform can also be calculated.

### 3.3.2 Correction of Thermal Drift

It is not always possible to fully compensate for the influence of thermal fluctuations by monitoring and managing the ambient conditions. This is especially the case for positioning axes with resolutions and accuracies down to the subnanometer range. Even the smallest change in temperature during the measurement procedure can cause changes in position due to thermal expansion of the measurement setup or measuring equipment. Also mechanical tension in the measurement setup and the resulting relaxation processes lead to such changes in position. If the highest level of accuracy is required, these effects can have a massive influence on the measurement results. Therefore, it may be necessary to correct for these thermal effects.

The measurement data is corrected based on the following assumptions:

- The positioning axis that is being tested does not itself change over time, except for the mechanical changes resulting from changes in temperature.
- If the temperature were stable, the first measurement point would always be measured at an identical position.
- Thermal drift is not position-dependent but rather time-dependent.

To make the correction, a straight line is calculated through the first measurement points of repeated sets of measurements taken one after another. This straight line (temporal) can then be subtracted from the measurement data for one set of measurements. This makes it possible to compensate for thermal drift.

The protocol indicates whether or not a correction has been carried out for thermal drift.

### 3.3.3 Linear Correction of Raw Data

If only the deviation from an ideal straight line is of interest for an evaluation, the raw data have to be initially corrected linearly. This means that a regression line is subtracted from the positioning error  $x_{ij}$  with respect to the commanded target position  $P_i$ . This is calculated according to the method of least squares.

The measurement protocol indicates whether or not a linear correction has been carried out.

## 4 Data Analysis

The calculation rules and the corresponding calculation formulae used to evaluate the measurement data are presented here.

### 4.1 Basic Data for the Positioning Error

Firstly, the unidirectional positioning error is determined at every measured target position for both directions  $x_i \uparrow$  and  $x_i \downarrow$ . This data is then used to calculate the mean positioning errors  $\bar{x}_i \uparrow$  and  $\bar{x}_i \downarrow$  for both directions at every individual position:

$$\bar{x}_i \uparrow = \frac{1}{n} \sum_{j=1}^n x_{ij} \uparrow$$

and

$$\bar{x}_i \downarrow = \frac{1}{n} \sum_{j=1}^n x_{ij} \downarrow$$

Using the mean positioning error at the points  $\bar{x}_i \uparrow$  and  $\bar{x}_i \downarrow$ , which only measure the two individual directions (forwards and backwards), the mean value is determined. This is the mean bidirectional positioning error at every individual position  $\bar{x}_i$ :

$$\bar{x}_i = \frac{\bar{x}_i \uparrow + \bar{x}_i \downarrow}{2}$$

The calculated values can be used to plot a mean curve that shows the mean errors of the axis across the entire travel range (see Figure 4).

The mean values show the systematic deviation. Other variables for a positioning axis can be attributed to statistical deviations. For this reason, the standard deviations at the respective target positions  $P_i$  are determined separately for the forwards and backwards directions. This is carried out using the following formulae:

$$s_{i \uparrow} = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (x_{ij \uparrow} - \bar{x}_{i \uparrow})^2}$$

and

$$s_{i \downarrow} = \sqrt{\frac{1}{n-1} \sum_{j=1}^n (x_{ij \downarrow} - \bar{x}_{i \downarrow})^2}$$

These five datasets together provide a good overview of the size of the statistical deviations and how the positioning axis behaves over the measuring range. Figure 4 shows an example overview of the complete data. Other variables can be calculated on this basis.

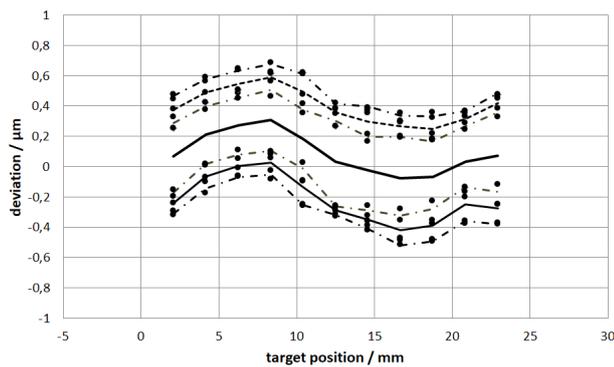


Fig. 4 The positioning error for an axis for both the forwards and backwards directions

## 4.2 The Mean Bidirectional Positioning Error M

The mean bidirectional positioning error  $M$  is the difference between the maximum and minimum values for the mean bidirectional positioning errors at the points  $\bar{x}_i$

$$M = \max(\bar{x}_i) - \min(\bar{x}_i)$$

The mean bidirectional positioning error  $M$  is the maximum deviation (peak-peak) of the mean curve as described above, meaning the mean positioning error of the axis.

This is shown in red in Figure 5.

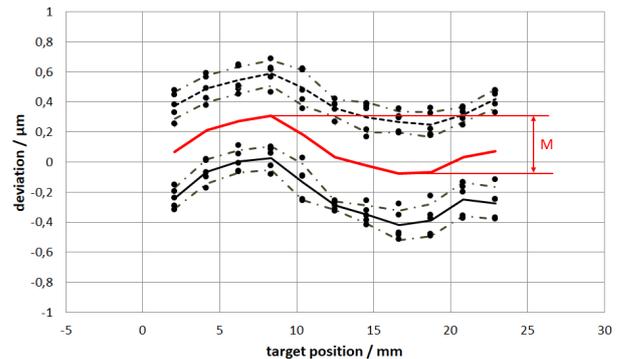


Fig. 5 The mean bidirectional positioning error  $M$

If a correction line (see 3.3.3) has been subtracted from the raw data when calculating  $M$ , this value is designated as  $M_L$ .

## 4.3 The Backlash B

To determine the size of the error between the positioning processes in the forwards and backwards directions, the maximum positioning backlash  $B$  is calculated. It represents the maximum deviation between the forwards and backwards directions.

$$B = \max [|\bar{x}_{i \uparrow} - \bar{x}_{i \downarrow}|]$$

This is shown in red in Figure 6.

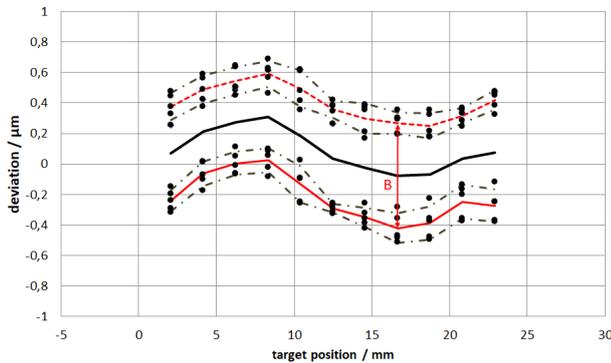


Fig. 6 The maximum positioning backlash  $B$

A mean backlash can also be determined on this basis. This indicates the size of the mean deviation between the forwards and backwards directions. To determine the mean positioning backlash  $\bar{B}$ , the sum of the mean values for the backlash at all individual positions is determined as follows:

$$\bar{B} = \left| \frac{1}{m} \sum_{j=1}^m (\bar{x}_i \uparrow - \bar{x}_i \downarrow) \right|$$

*The backlash  $B$  indicates how the positioning axis behaves if the direction is reversed.*

#### 4.4 Unidirectional and Bidirectional Repeatability $R$

Another important characteristic of a positioning device is repeatability.

To calculate the repeatability, the bidirectional positioning repeatability  $R_i$  is first calculated for one position. It is calculated as follows:

$$R_i = \max[s_i \uparrow + s_i \downarrow + |B_i|; 2s_i \uparrow; 2s_i \downarrow]$$

with

$$B_i = \bar{x}_i \uparrow - \bar{x}_i \downarrow$$

*The bidirectional repeatability indicates how "well" each individual target position will always be "hit" from different directions.*

Accordingly, the maximum bidirectional repeatability is the maximum value of the repeatability values at the target positions:

$$R = \max [R_i]$$

This is shown in red in Figure 7.

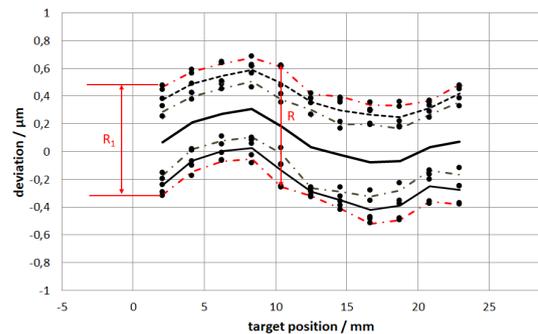


Fig. 7 The maximum bidirectional repeatability  $R$ , as well as, for example,  $R_1$  for the first target position

The maximum unidirectional positioning repeatability  $R \downarrow \uparrow$  is the maximum value of the standard deviations of the unidirectional repeatability for a positioning axis:

$$R \downarrow \uparrow = \pm \max [s_i \uparrow; s_i \downarrow]$$

*The maximum unidirectional repeatability indicates how large the maximum deviation at any target position can be when approaching the position from the same direction.*

The value for the unidirectional positioning repeatability is given as a  $\pm$  value.

The average unidirectional positioning repeatability  $\bar{R} \downarrow \uparrow$  is calculated as follows:

$$\bar{R} \downarrow \uparrow = \pm \frac{\frac{1}{m} \sum_{i=1}^m s_i \uparrow + \frac{1}{m} \sum_{i=1}^m s_i \downarrow}{2}$$

It is the average of the unidirectional repeatabilities in both directions.

## 4.5 The Bidirectional Positioning Accuracy A

*The bidirectional positioning accuracy A is a measurand that combines all of the errors that arise and thus represents the maximum error that can occur during positioning.*

The bidirectional positioning accuracy is the range that is calculated by combining the bidirectional systematic deviations with the statistical deviations:

$$A = \max [\bar{x}_i \uparrow + s_i \uparrow; \bar{x}_i \downarrow + s_i \downarrow] - \min [\bar{x}_i \uparrow - s_i \uparrow; \bar{x}_i \downarrow - s_i \downarrow]$$

This is shown in red in Figure 8.

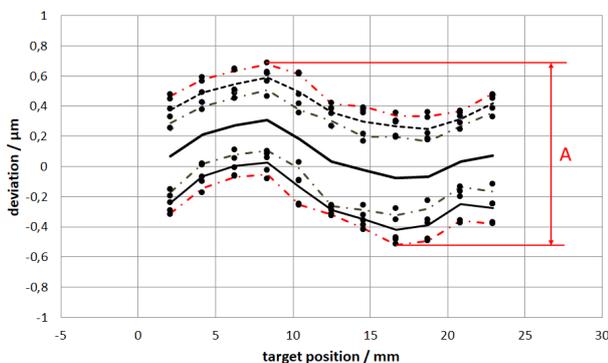


Fig. 8 The bidirectional positioning accuracy A

To ensure the consistency of the bidirectional repeatability, the calculation for the bidirectional positioning accuracy uses only one standard deviation and not two standard deviations.

## 4.6 The Stroke S

The testing of a positioning axis shall prove that the entire travel range can be used, which means that the axis being tested can reach every numerically controlled target position if the travel range was also selected as the measuring range. For this purpose, the mean values for the positions at the end points of the measuring range are determined. The difference between these values is the stroke S of the positioning axis.

## 4.7 The Linearity L

*The linearity describes the maximum deviation from the measured values for an ideal straight line.*

To calculate the linearity, the measured values at the target positions  $P_0$  to  $P_{m+1}$  are used. A straight line is taken that passes through the end points of the measuring range and is subtracted from the calculated deviations. These calculated errors are then designated  $xl_{ij}$ .

Accordingly, the positioning linearity L is calculated as follows:

$$L = \max \left[ \left| \frac{1}{n} \sum_{j=1}^n xl_{ij} \uparrow \right| \right]$$

This is shown in red in Figure 9.

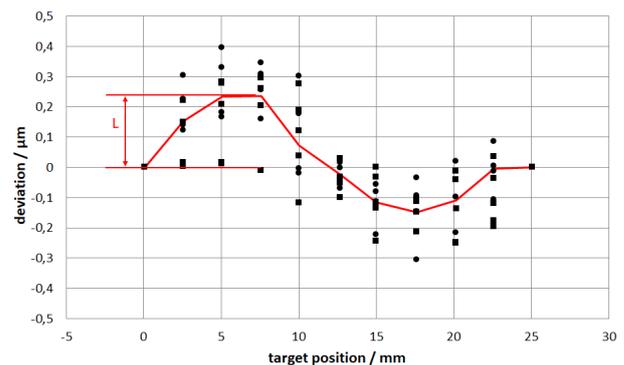


Fig. 9 Determination of the positioning linearity L

This can be used to calculate the percentage linearity  $L_p$  (in percent) where  $L_m$  denotes the measuring range:

$$L_p = 100 \cdot \frac{L}{L_m}$$

## 4.8 The Hysteresis H

The hysteresis  $H$  of an axis is the maximum difference of forwards and backwards values at all target positions:

$$H = \max \left[ \left| \frac{1}{n} \sum_{j=1}^n x_{l_{ij}} \uparrow - \frac{1}{n} \sum_{j=1}^n x_{l_{ij}} \downarrow \right| \right]$$

The calculation of  $H$  is carried out in the same way as the calculation of the backlash  $B$ .

Accordingly, the percentage hysteresis  $H_p$  (in percent) is calculated as follows

$$H_p = 100 \cdot \frac{H}{L_m}$$

## 5 Annex 1: The Coordinate Systems Used and Designations of the Axes of Motion

In general, PI uses a right-handed coordinate system to designate the axes of motion. The linear axes are thus labeled with X, Y, and Z and the rotational axes with A, B, and C. The rotational axes are assigned according to the linear axes, e.g. A is the rotational axis around the linear axis X.

In places where further clarification is necessary, the rotational axes are also labeled with an explicit reference to the linear axes, e.g. A (rot X). To illustrate how the axes are designated, the coordinate system is once again shown in Figure 10.

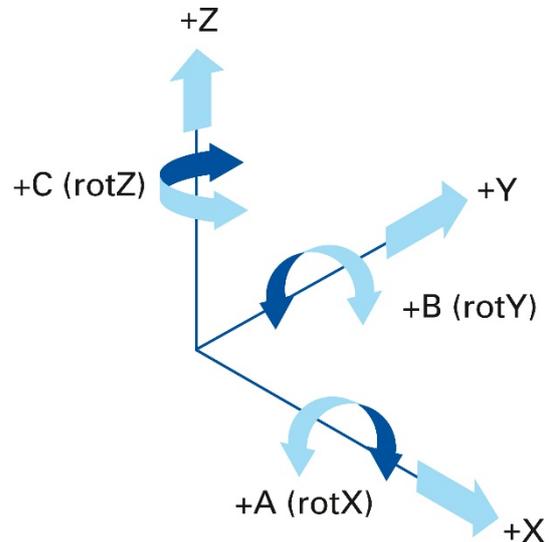


Fig. 10 Right-handed coordinate system

## 6 Annex 2: Mechanical Crosstalk of a Movement and Designation of the Error

In the case of mechanical positioning axes, it is usually not just the positioning error in the direction of motion that is relevant but also the mechanical crosstalk of a linear motion in the other linear and rotational directions. To determine this mechanical crosstalk, linear measurements orthogonal to the direction of motion and corresponding angle measurements are carried out. Here, a measurement is carried out according to the motion profile of the measurement procedure described in chapter 3.1. The same methods as for the positional data are used to analyze these data. However, the analysis does not subtract the actual position from the target position but uses the raw data.

Straightness and flatness are calculated from the mean bidirectional positioning error after a linear correction  $M_L$  has been carried out.

It is still important to note here that the rotational error can transfer to a translational error at the functional point in certain circumstances, depending on where the zero point for the parasitic tilt lies.

The errors themselves have a clear nomenclature. The various measurable errors for linear motion are shown in Figure 11.

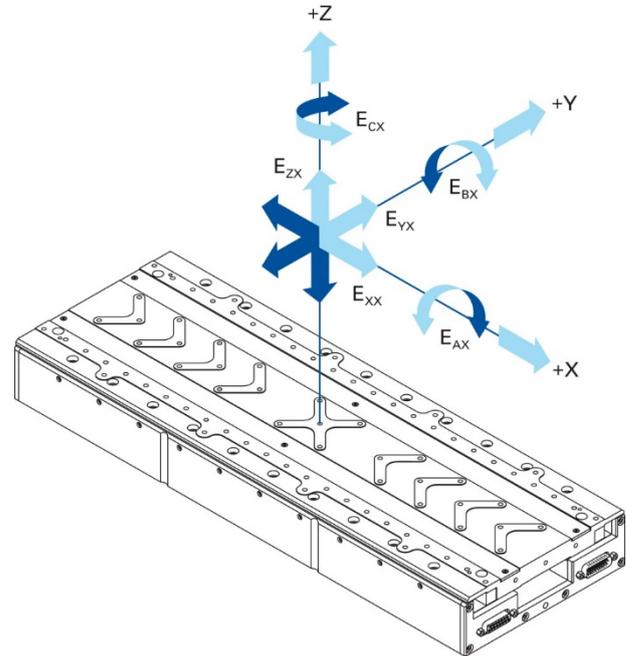


Fig. 11 Nomenclature of errors in the various axes for a linear motion.

The nomenclature used at PI for these different errors is thus as follows:

$E_{<Axis\ designation\ where\ the\ error\ has\ been\ measured><commanded\ axis>}$

$X$  Axis of the commanded linear motion

$E_{AX}$  Angular error around the X-axis (roll)

$E_{BX}$  Angular error around the Y-axis (pitch)

$E_{CX}$  Angular error around the Z-axis (yaw)

$E_{XX}$  Positioning error of the X-axis

$E_{XXL}$  Positioning error of the X-axis, minus regression line

$E_{YX}$  Straightness error in the direction of the Y-axis (straightness)

$E_{ZX}$  Straightness error in the direction of the Z-axis (flatness)

If two axes are moving (e.g. X and Y), it is still possible for a combined parasitic motion to occur in the Z-axis. This error is called the out-of-plane error.

Similar errors can also be found on rotational positioning axes. Accordingly, Figure 12 shows the various measurable errors for rotational motion.

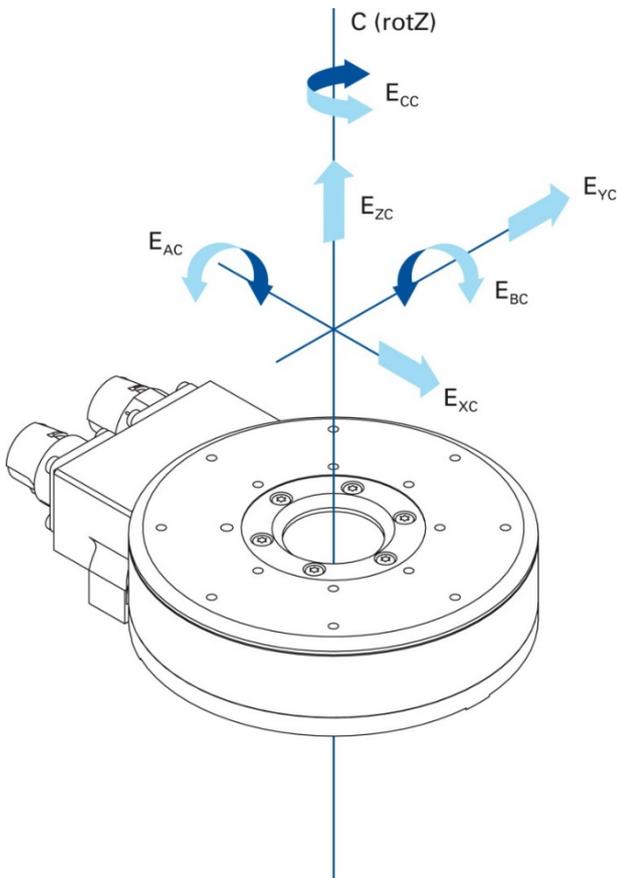


Fig. 12 Nomenclature of errors in the various axes for a rotary motion.

$E_{XC}$  Radial error in the direction of the X-axis  
 $E_{YC}$  Radial error in the direction of the Y-axis  
 $E_{ZC}$  Axial error from C

$E_{AC}$  Tilt error around the X-axis  
 $E_{BC}$  Tilt error around the Y-axis  
 $E_{CC}$  Angular positioning error of C

A combined rotatory error, i.e. a radial error motion, is described as follows:

$E_{XYC}$  Radial error of C

Accordingly, a combined tilt deviation is referred to as follows:

$E_{ABC}$  Tilt error of C

## 7 Annex 3: Example Measurement Protocol

The data from the measurements and calculations can be accessed by outputting them to a measurement protocol. An example protocol is shown in Figure 13. The protocol is split into different sections:

1. General description, e.g. of the system components and parameters
2. Description of the measurement, e.g. axes, measuring range, measuring parameters, corrections
3. Measurement results, e.g. the linear deviations
4. Measurement results, e.g. the angular deviations
5. Graphical representation of the measurement results

The calculation rules for the measurement values given in 3. and 4. are described in chapter 4 and can thus be derived from the stated formulae. This applies for both linear deviations and angular deviations.

## Test Protocol for Linear Stage H-811K097\_AXIS\_Z

File: H-811K097\_AXIS\_Z\_118069051\_EZZ\_FullRange\_Vergleich\_ztz.pdf  
 Protocol no.: FM 7.5-79\_A1  
 2020-05-14 10:07:20, Script version: 1.20.20.0, Signed on 2020-05-13 15:31:23 for Framework 1.20.20.0

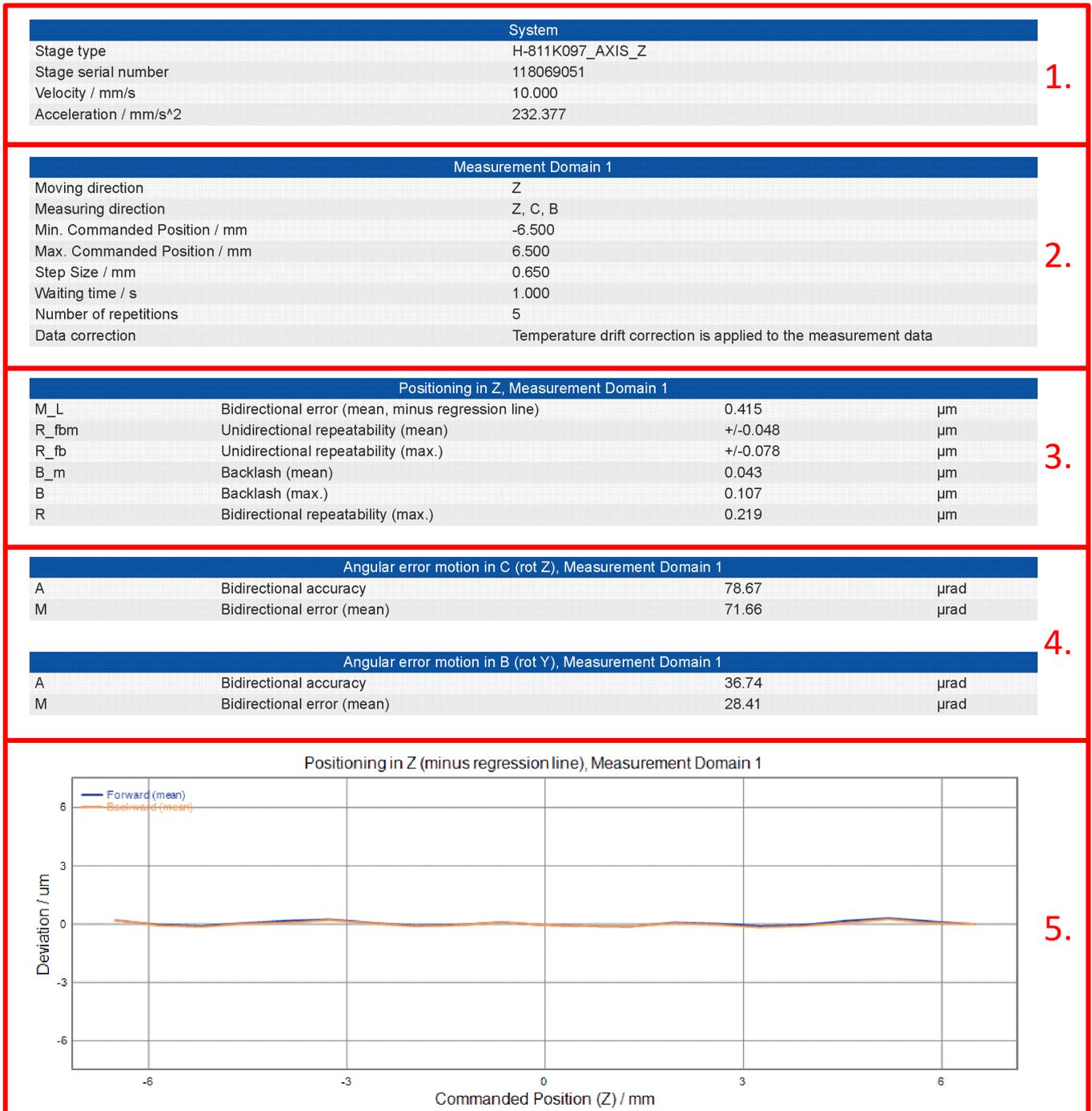


Fig. 13 Example measurement protocol

## 8 Other Documents and Links

Whitepaper This is how PI Does Measuring - Part I

[https://www.physikinstrumente.de/fileadmin/user\\_upload/physik\\_instrumente/files/WP/PI-WP4011D-Messtechnik-Messumgebung.pdf](https://www.physikinstrumente.de/fileadmin/user_upload/physik_instrumente/files/WP/PI-WP4011D-Messtechnik-Messumgebung.pdf)

ISO 230-1

<https://www.din.de/de/mitwirken/normenausschuesse/nwm/normen/wdc-beuth:din21:151552174>

ISO 230-2

<https://www.din.de/de/mitwirken/normenausschuesse/nwm/normen/wdc-beuth:din21:206720855>

ASME B.5.54 - 2005

<https://www.asme.org/codes-standards/find-codes-standards/b5-54-methods-performance-evaluation-computer-numerically-controlled-machining-centers>

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## PI in Brief

For many years, PI (Physik Instrumente), founded in 1970, has been a market and technology leader for high-precision positioning technology and piezo applications in the semiconductor industry, life sciences; photonics, and in industrial automation. In close cooperation with customers from all over the world and for 50 years now, PI's specialists (approx. 1,300) have been pushing, again and again, the boundaries of what is technically possible and developing customized solutions from scratch. Technologies from PI achieve reproducible accuracies in the millionth of a millimeter range. More than 350 granted and registered patents underline the company's claim to innovation.

PI develops, manufactures, and qualifies all core technologies in-house, thereby constantly setting new standards for precision positioning: Piezoceramic patch transducers and actuators, electromagnetic drives, and sensors working in the nanometer range. As the majority owner of ACS Motion Control, PI is also a leading global manufacturer of modular motion control systems for multi-axis drive systems and develops customized complete systems for industrial applications with the highest precision and dynamics.

With six manufacturing sites and 15 sales and service offices in Europe, North America, and Asia, PI is represented wherever high-tech solutions are developed and manufactured.